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NONNATURAL VIBRATIONS OF HYDRAULIC SHOCK-ABSORBERS

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Based on the procedure developed by the present authors, calculation of a construction unit intended for reducing high levels of vibrations has been carried out. The corresponding physical characteristics of the unit elements have been selected. For a multi-element model, an equivalent of a change-over to a single-mass vibrational system has been found. A criterion for selecting the physical and geometric parameters of vibroshock-absorbers in the form of an inequality between the coefficients given has been obtained.

Introduction. An inseparable part of the development of the current automotive industry is the application of new components of vibration isolation. Among them, the development of passive uniaxial vibroshock-absorbers occupies a leading place and is preferable compared to active ones based on cost, ease of fabrication, economy, and interchangeability. They do not require additional cost for electronic devices to support electric and magnetic fields in active hydraulic supports.

Passive vibration isolators of mobile facilities are rubber-metal vibroshock-absorbers, silent-blocks, hydraulic supports, etc. Predesigning of each objects is connected with the use of the simplest elastic and damping elements depending linearly on displacements and velocities.

Statement of the Problem. A general model of vibration isolation (Fig. 1) involving a Russian hydraulic support [1] is represented by a dynamic scheme (Fig. 2), with the vibrations of a passive uniaxial hydraulic support with an inertial transformer being the vibrations of a two-mass mechanical system, where the mass m_0 is infinitely large and comparable with the base [2]. The vibrational periodically varying force $F(t) = A \sin(\omega t)$ applied to m_0 causes kinematic excitations of the base x_0, $\dot{x}_0$, and $\ddot{x}_0$ that are easily determined experimentally:

$$\ddot{x}_1 = b_{10}\dot{x}_0 - b_{11}\dot{x}_1 + b_{12}\dot{x}_2 + c_{10}x_0 - c_{11}x_1 + c_{12}x_2 + f_1 \,, \tag{1}$$

$$\ddot{x}_2 = b_{20}\dot{x}_0 + b_{21}\dot{x}_1 - b_{22}\dot{x}_2 + c_{20}x_0 + c_{21}x_1 - c_{22}x_2 + f_2 \,. \tag{2}$$

With differentiable right-hand sides of system (1), (2) with harmonic trigonometric functions for the problems of vibration isolation the following disconnected system of two fourth-order differential equations is obtained [2]:

$$x_1^{\mathrm{IV}} + \Delta_3 x_1''' + \Delta_2 x_1'' + \Delta_1 x_1' + \Delta_0 x_1 = f_1'' + \Delta_{f'}^2 + \Delta_f^2 \,, \tag{3}$$

$$x_2^{\mathrm{IV}} + \Delta_3 x_2''' + \Delta_2 x_2'' + \Delta_1 x_2' + \Delta_0 x_2 = f_2'' - \Delta_{f'}^1 - \Delta_f^1 \,. \tag{4}$$

Here

$$\Delta_3 = \begin{vmatrix} b_{11} & -1 \\ b_{22} & 1 \end{vmatrix} = (b_{11} + b_{22}) \,, \quad \Delta_2 = \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} + \begin{vmatrix} c_{11} & -1 \\ c_{22} & 1 \end{vmatrix} \,, \quad \Delta_1 = \begin{vmatrix} c_{11} & c_{12} \\ b_{21} & b_{22} \end{vmatrix} + \begin{vmatrix} b_{11} & b_{12} \\ c_{21} & c_{22} \end{vmatrix} \,,$$

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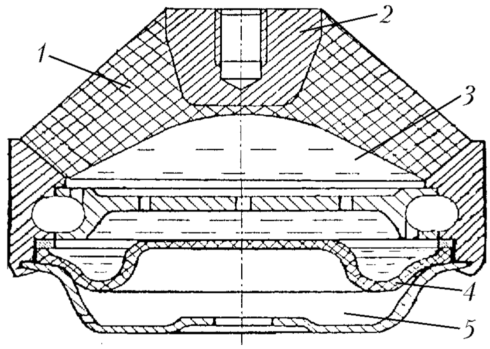

Fig. 1. Hydraulic support with an inertial channel and separating membrane: 1) rubber; 2) metal; 3) liquid; 4) membrane; 5) air.

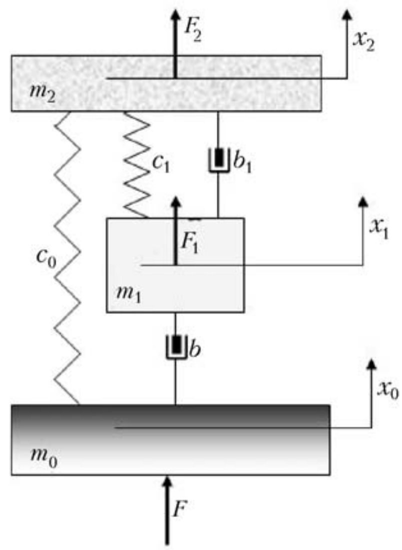

Fig. 2. General dynamic scheme of the hydraulic support with an inertial transformer.

$$\Delta_0 = \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix}, \quad \Delta_{f'}^2 = \begin{vmatrix} b_{12} & f_1' \\ -b_{22} & f_2' \end{vmatrix}, \quad \Delta_f^2 = \begin{vmatrix} c_{12} & f_1 \\ -c_{22} & f_2 \end{vmatrix}, \quad \Delta_{f'}^1 = \begin{vmatrix} -b_{11} & f_1' \\ b_{21} & f_2' \end{vmatrix}, \quad \Delta_f^1 = \begin{vmatrix} -c_{11} & f_1 \\ c_{21} & f_2 \end{vmatrix}.$$

The characteristic equations for system (3), (4) are identical, since the dynamic mechanical system possesses its own unique set of physical characteristics. The right-hand sides Eqs. (3), (4) differ and describe forced vibrations of the mechanical system. The natural vibrations are distinguished by the assigned initial conditioned. The general characteristic equation has the form

$$\lambda^4 + \Delta_3\lambda^3 + \Delta_2\lambda^2 + \Delta_1\lambda + \Delta_0 = 0\,. \tag{5}$$

For the dynamic scheme of the vibrations of the hydraulic shock-absorber and engine of a tractor (Fig. 2) we will select the following physical parameters: a = 1 N/kg, m_1 = 0.08 kg, m_2 = 70 kg, $b = 2\cdot10^4$ N·sec/m, $b_1 = 0.5\cdot10^4$ N·sec/m, c_0 = 23,000 N/m, c = 2000 N/m, and ω = 62.8 rad/sec. In the system of vibration isolation (Fig. 2) m_0 stands for a rigid base with the mass exceeding in a physical equivalent the remaining parameters. If $m_0 \to \infty$, then the vibrational acceleration will have the form $\ddot{x}_0 = a \sin \omega t$, the vibration velocity will be $\dot{x}_0 = -(a/\omega) \cos \omega t$, and the vibrational displacement will be $x_0 = -(a/\omega^2) \sin \omega t$. The coefficients of the system of equations of motion (1) will take the values b_{10} = 250,000.0, b_{11} = 312,500.0, b_{12} = 62,000.0, b_{20} = 0, b_{21} = 71.43, b_{22} = 71.43, c_{10} = 0, c_{11} = 25,000.0,

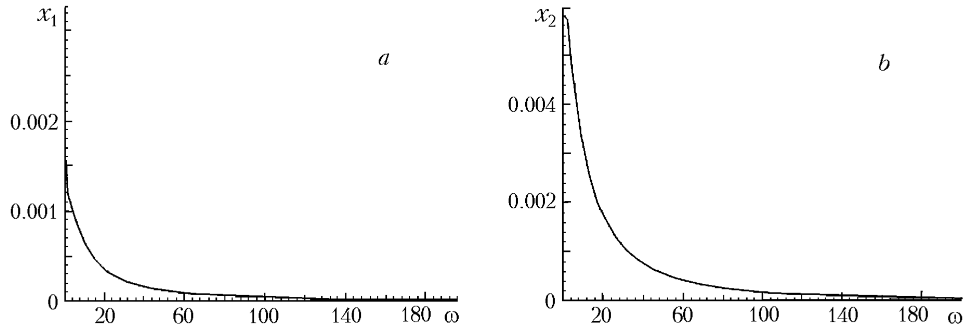

Fig. 3. Amplitude-frequency characteristics of displacements x_1 and x_2.

c_{12} = 25,000.0, c_{20} = 328.57, c_{21} = 28.57, c_{22} = 357.14, $f(t)$ = sin ωt, f_1 = 10.0 sin ηt, and f_2 = –2.0 cos μt. Two variants are possible: with a given external effect F and without it; in variant I the amplitude of the force $F = 0$ and in variant II the amplitude of force F is equal to the unit force. Substituting the found coefficients into the system of equations (1), (2), we obtain

$$\ddot{x}_1 = -312{,}500.0\dot{x}_1 + 62{,}500.0\dot{x}_2 - 25{,}000.0x_1 + 25{,}000.0x_2 + \mathcal{F}_1 ,$$

$$\ddot{x}_2 = 71.43\dot{x}_1 - 71.43\dot{x}_2 + 28.57x_1 - 357.14x_2 + \mathcal{F}_2 . \tag{6}$$

Here $\mathcal{F}_1$ = 10.0 sin ηt, $\mathcal{F}_2$ = –2.0 cos μt for variant I and $\mathcal{F}_1$ = 10.0 sin $\eta t - 250{,}000.9$ cos $(\omega t)/\omega$, $\mathcal{F}_2$ = –2.0 cos $\mu t - 328.6$ sin $(\omega t)/\omega^2$ for variant II.

Numerical-Analytical Solution of the Problem. *Variant I. Calculation of the dynamic characteristics of the hydraulic support with the action of a force and without allowance for the kinematic excitation of the base.* Dividing the variables in (6), we arrive at a new system of linear inhomogeneous differential equations of fourth order with constant coefficients:

$$x_1^{IV} + 312{,}571.4x_1''' + 17{,}882{,}500.0x_1'' + 109{,}821{,}428.5x_1' + 8{,}214{,}285.7x_1 = (-10.0\eta^2 + 3571.4)\sin(\eta t)$$
$$+ 714.3\eta\cos(\eta t) + 125{,}000.0\mu\sin(\mu t) - 50{,}000.0\cos(\mu t) ,$$
$$\tag{7}$$
$$x_2^{IV} + 312{,}571.4x_2''' + 17{,}882{,}500.0x_2'' + 109{,}821{,}428.5x_2' + 8{,}214{,}285.7x_2 = (2.0\mu^2 - 50{,}000.0)\cos(\mu t)$$
$$+ 625{,}000.0\mu\sin(\mu t) + 714.3\eta\cos(\eta t) + 285.7\sin(\eta t) .$$

By the form of the characteristic polynomial $Q(\lambda) = 8{,}214{,}285.7 + 109{,}821{,}428.5\lambda + 17{,}882{,}500.0\lambda^2 + 312{,}571.4\lambda^3 + \lambda^4$ of the given problem one is capable of judging whether or not the given mechanical system processes stability, stability with a margin, and quality. The stability parameters are determined as follows: s_1 = 0.45934, s_2 = 351.3482, s_3 = 17,882,500.0, Ω_1 = 82.1062, Ω_2 = 9.31579, Ω_3 = 5463.4921, W_1 = 764.8852, and W_2 = 50896.7935. The necessary conditions of the system stability $s_1 < s_2 < s_3$ and sufficient conditions $W_1 > 1$, $W_2 > 1$ are fulfilled; moreover, the vibrations of the vibration damper with viscous friction presented in Fig. 2 have a stability and quality margin $W_1 > 3$, $W_2 > 3$, $\Omega_1 > 4$, $\Omega_2 > 4$, and $\Omega_3 > 4$.

The hydraulic support with inertial transformer possesses the stability and quality margin and has no resonance regimes. Its amplitude-frequency characteristics for x_1 and x_2 are presented in Fig. 3 [3].

The solution of the system of equations (7) at ω = 34.2 rad/sec, η = 15.7 rad/sec, and μ = 5.7 rad/sec is given by

$$x_1 = 0.4598955754\cdot10^{-7}\sin(\eta t) - 0.2543564254\cdot10^{-5}\cos(\eta t) - 0.1055089276\cdot10^{-3}\sin(\mu t)$$
$$+ 0.7146466721\cdot10^{-4}\cos(\mu t) ,$$

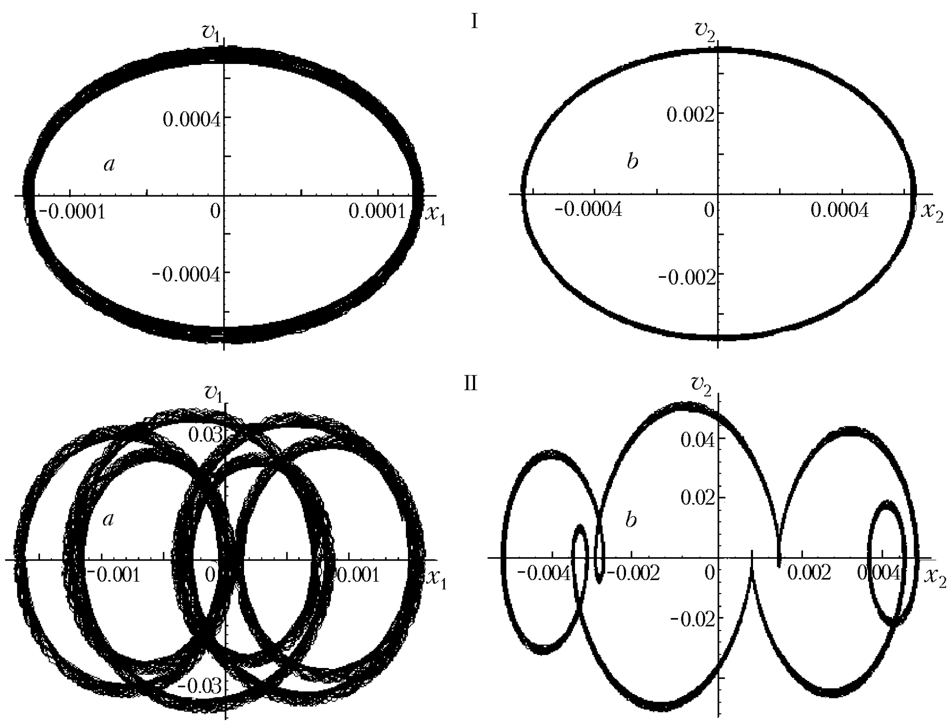

Fig. 4. Dependence of the velocities v_1 of displacement x_1 (a) and of v_2 on x_2 (b) for variants I and II.

$$x_2 = -0.5300657375 \cdot 10^{-3} \sin(\mu t) + 0.3535301547 \cdot 10^{-3} \cos(\mu t) + 0.2300382143 \cdot 10^{-6} \sin(\eta t)$$

$$- 0.2522037785 \cdot 10^{-5} \cos(\eta t) . \tag{8}$$

The phase trajectories of masses m_1 and m_2 in the coordinates (v_1, x_1) and (v_2, x_2) are given in Fig. 4. The center of the coordinate plane $x_1 = 0$, $v_1 = 0$, and $x_2 = 0$, $v_2 = 0$ is the position of equilibrium.

Variant II. Calculation of the dynamic characteristics of the hydraulic support with force action and kinematic excitation of the base. Equilibrium equations (1), (2) will take the form

$$x_1^{IV} + 312{,}571.4 x_1''' + 17{,}882{,}500.0 x_1'' + 109{,}821{,}428.5 x_1' + 8{,}214{,}285.7 x_1 = -((10.0\omega^2\eta^2$$

$$- 3571.428571\omega^2) \sin(\eta t) - 714.3\eta\omega^2 \cos(\eta t) + (0.1098214286 \cdot 10^9 \omega - 250{,}000.0\omega^3) \cos(\omega t)$$

$$+ 125{,}000.0\mu\omega^2 \cos(\mu t) + 50{,}000.0\omega^2 \sin(\mu t) + (0.8214285715 \cdot 10^7 - 0.1785714286 \cdot 10^8 \omega^2) \sin(\omega t))/\omega^2 ,$$

$$x_2^{IV} + 312{,}571.4 x_2''' + 17{,}882{,}500.0 x_2'' + 109{,}821{,}428.5 x_2' + 8{,}214{,}285.7 x_2 = -((-2.0\omega^2\mu^2$$

$$+ 50{,}000.0\omega^2) \sin(\mu t) + (0.8214285715 \cdot 10^7 - 0.1785747143 \cdot 10^8 \omega^2) \sin(\omega t) + 625{,}000.0\mu\omega^2 \cos(\mu t)$$

$$+ 0.1098214285 \cdot 10^9 \omega \cos(\omega t) - 714.3\eta\omega^2 \cos(\eta t) - 285.7142857\omega^2 \sin(\eta t))/\omega^2 . \tag{9}$$

The general characteristic equation coincides with variant I. Therefore the given hydraulic support with an inertial transformer but with other conditions of kinematic excitation possesses the stability and quality margin and has no resonance regimes like in the previous case.

The solution of the system of equations (6) at ω = 34.2 rad/sec, η = 15.7 rad/sec, and μ = 5.7 rad/sec is given by

$$x_1 = 0.4599100868\cdot 10^{-7}\sin(15.7t) - 0.2543563969\cdot 10^{-5}\cos(15.7t) + 0.6707181607\cdot 10^{-3}\cos(5.7t)$$

$$- 0.5779326765\cdot 10^{-3}\sin(5.7t) - 0.8175324805\cdot 10^{-3}\sin(34.2t) + 0.8669606163\cdot 10^{-4}\cos(34.2t)\,,$$

$$x_2 = - 0.3065741176\cdot 10^{-2}\sin(5.7t) + 0.3178955044\cdot 10^{-2}\cos(5.7t) + 0.4347359661\cdot 10^{-3}\cos(34.2t)$$

$$- 0.6719247998\cdot 10^{-3}\sin(34.2t) + 0.2300371112\cdot 10^{-6}\sin(15.7t) - 0.2522037989\cdot 10^{-5}\cos(15.7t)\,. \tag{10}$$

The phase trajectories of masses m_1 and m_2 in the coordinates (v_1, x_1), (v_2, x_2) are presented in Fig. 4. The center of the coordinate phase plane x_1 = 0, v_1 = 0 and x_2 = 0, v_2 = 0 is the position of equilibrium. With increase in the angular velocity ω, the amplitudes of vibrations of masses m_1 and m_2 decrease sharply and do not have resonances. To investigate the dynamic characteristics of the hydraulic supports used as the suspensions of the power-generating sets of automobiles the methods of amplitude-frequency characteristics apply well: they allow one to determine the dynamic coefficients of rigidity and the coefficients of the transmission of loadings.

The choice of the rational parameters of the hydraulic support in the form of a single-mass elastically damping shock-absorber is an essential factor in ensuring stable, extra stable, qualitative vibrations without sharp resonances of a test object. In this case, a uniaxial hydraulic support of arbitrary form with many masses can be reduced by physical parameters to a dynamic model with one mass, as is done by German companies "Simrit," "Bosch," "Freudenberg," "Metneller," etc. that produce hydraulic supports. The physical and geometric parameters of hydraulic supports are selected by the criteria of the stability and quality margins:

$$b > 2\sqrt{cm} \tag{11}$$

for a known given general rigidity c and general damping coefficient b determined by impedance method [4]:

$$c = \frac{\omega\,(b^2\,(m_1\omega^2\,(-c_1^2 - b_1^2\omega^2 + c_1 m_1\omega^2) + c_0\,(c_1^2 + b_1^2\omega^2 - 2c_1 m_1\omega^2 + m_1^2\omega^4)))}{2bb_1 m_1^2\omega^5 + m_1^2\omega^3\,(c_1^2 + b_1^2\omega^2) + b^2\omega\,(c_1^2 + b_1^2\omega^2 - 2c_1 m_1\omega^2 + m_1^2\omega^4)}$$

$$+ \frac{\omega\,(2bb_1 c_0 m_1^2\omega^4 + c_0 m_1^2\omega^2\,(c_1^2 + b_1^2\omega^2))}{2bb_1 m_1^2\omega^5 + m_1^2\omega^3\,(c_1^2 + b_1^2\omega^2) + b^2\omega\,(c_1^2 + b_1^2\omega^2 - 2c_1 m_1\omega^2 + m_1^2\omega^4)}\,,$$

$$b = \frac{bc_1^2 m_1^2\omega^3 + b^2 b_1 m_1^2\omega^5 + bb_1^2 m_1^2\omega^5}{2bb_1 m_1^2\omega^5 + m_1^2\omega^3\,(c_1^2 + b_1^2\omega^2) + b^2\omega\,(c_1^2 + b_1^2\omega^2 - 2c_1 m_1\omega^2 + m_1^2\omega^4)}\,,\quad m = m_2\,.$$

Substituting the known physical parameters into inequality (11), we ascertain the validity of the latter.

Conclusions. For the considered mechanical system "base–hydraulic support–engine" physical parameters are selected in such a way that no resonance regimes could be observed in it and that stable, extra stable, and qualitative vibrations could be realized.

NOTATION

A, a, amplitudes of vibrations of the external force related to mass m_0, N and N/kg; b, reduced rigidity, kg·sec^{-1}; b_1, b_2, coefficients of damping elements, kg·sec^{-1}; $b_{10} = b/m_1$, $b_{11} = (b_1 + b)/m_1$, $b_{12} = b_1/m_1$, $b_{20} = 0$, $b_{21} = b_1/m_2$, $b_{22} = b_1/m_2$, b_{10}, b_{11}, b_{12}, b_{20}, b_{21}, b_{22}, intermediate coefficients, sec^{-1}; c, reduced elasticity, kg·sec^{-2}; c_0, c_1, c_2, coefficients of the elasticity of springs, kg·sec^{-2}; $c_{10} = 0$, $c_{11} = -c_1/m_1$, $c_{12} = c_1/m_1$, $c_{20} = c_0/m_2$, $c_{21} = c_1/m_2$, $c_{22} = (c_0 + c_1)/m_2$, c_{10}, c_{11}, c_{12}, c_{20}, c_{21}, c_{22}, intermediate coefficients, sec^{-2}; F, F_1, F_2, external forces, N; f_1 =

F_1/m_1; $f_2 = F_2/m_2$; $_1$, $_2$, vibrational accelerations reduced to corresponding masses, m/sec^{-2}; m, reduced mass, kg; m_0, m_1, m_2, masses of the element of vibration isolator, kg; t, time, sec; v_1, v_2, velocities, m/sec; W_1, W_2, parameters of stability and quality according to Voronov; x_0, vibration displacement, m; $\dot{x}_0$, vibration velocity, m/sec; $\ddot{x}_0$, vibration acceleration, m/sec^2; x_1, x_2, displacements of bodies with masses m_1 and m_2, m; $\dot{x}_1$, $\dot{x}_2$, velocities, m/sec; $\ddot{x}_1$, $\ddot{x}_2$, accelerations, m/sec^2; λ, Δ, roots and coefficients of the characteristic equation of $Q(\lambda)$; η, μ, ω, frequencies of vibrations of external forces F_1, F_2, F, rad/sec^{-1}; Ω_1, Ω_2, Ω_3, stability parameters according to Voronov.

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